DOCUMENT RESUME

ED 116 728 JC 760 046

AUTHOR Finch, Harold L.; Tatham, Elaine L.
TITLE Declining Enrollments: Why? What Are the Trends and Implications?; and Enrollment Forecasting Techniques.
INSTITUTION Johnson County Community Coll., Overland Park, Kans.
PUB DATE 24 Sep 75
NOTE 39p.; Paper presented at the Meeting on Coping with Declining Enrollments (Hyannis, Massachusetts, September 24, 1975). Appendix A (sample printouts) may reproduce poorly

EDRS PRICE MF-$0.76 HC-$1.95 Plus Postage
DESCRIPTORS Birth Rate; Community Zoning; Computer Programs; Death; *Demography; *Educational Planning; *Enrollment Projections; *Mathematical Models; Migration Patterns; Multiple Regression Analysis; Research Methodology; Residential Patterns
IDENTIFIERS Cohort Survival

ABSTRACT

This document presents a modified cohort survival model which can be of use in making enrollment projections. The model begins by analytically profiling an area's residents. Each person's demographic characteristics--sex, age, place of residence--are recorded in the computer memory. Four major input variables are then incorporated into the model: (1) death rate--deaths are projected for males and females as a function of the number of people for each age; (2) birth rate--three equations are offered, with the most popular projecting the number of births by using the fertility rates of women by age; (3) net migration--total subarea net migration (the number of people moving in minus the number of people moving out of a given area) is assigned by age and sex; (4) land use--designations are based on zoning. Categories employed in this model are: special, agricultural, industrial, commercial, residential (low, medium, and high density), and unzoned. Zoning is specified for the beginning year, and the user has the option of incorporating anticipated zoning changes into the model. Mathematical and computer procedures used in this model include regression analysis and the two-point logistic curve. A hypothetical application of the model is presented, and practice problems are proposed. Sample printouts are appended. (NHM)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

ED116728

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY.

PREFACE

This report contains assorted material abstracted from presentations to the AASA-NASE Conference on Declining Enrollments. Dr. Harold L. Finch is Interim Chief Administrative Officer and Vice President of Johnson County Community College; Dr. Elaine L. Tatham is Director of Institutional Research of the college. Dr. Finch made the presentations.

TABLE OF CONTENTS

	Page
PREFACE	1
I. THE EDUCATIONAL NEED FOR DEMOGRAPHIC PLANNING	1
II. A CASE STUDY IN DEMOGRAPHIC ANALYSIS AND PLANNING	2
Approach: Analytical Modeling	3
Simulation Process	4
"What If" Variables	6
Modified Cohort Analysis	9
Output	9
Hypothetical Illustration	13
Illustration of Results	13
III. MISCELLANEOUS THEORY	15
Two-Point Linear Regression	16
Multiple-Point Linear Regression	17
Two-Point Logistic Curve	18
Multiple-Point Logistic Curve	20
IV. PRACTICE PROBLEMS	22
Practice Problem: Regression and Cohort Survival Methodology	23
Practice Problem: Two-Point Logistic Method	25
APPENDIX A. SAMPLE PRINTOUTS	27

I. THE EDUCATIONAL NEED FOR DEMOGRAPHIC PLANNING

As the products of the postwar baby boom reached their late teens in the 1960's, it was inevitable that high school enrollments would increase at unprecedented rates. Likewise, the baby bust which followed the boom was destined to have an equally dramatic--but opposite--affect in the 1970's. The impact is already being felt. For those institutions that are unprepared and do not begin to prepare, the worst may yet lie ahead. For the prepared, the next several years will present unprecedented challenges--opportunities for self-renewal and revitalization that will put them in good stead to survive the 70's and to resume vigorous growth in numbers and efficiency in the 80's and on into the twenty-first century.

Some important lessons can be learned from school districts that have been hard hit by declining enrollments. In examining a number of these cases, there seems to be a common pattern of events and circumstances the precede enrollment crises:

• *Failure to Recognize the Problem.* Enrollment forecasts are consistently high. Projections tend to perpetuate the heyday trends of the last decade and do not properly take into account key demographic factors. Missed forecasts are explained away as one-time flukes. Having diagnosed the problem incorrectly, staffing and budgeting plans for the next year then tend to be projected on the basis that enrollments and revenues will be "back to normal."

• *Too Little Too Late.* The seriousness of the problem is recognized too late. The dynamics of the situation are very much like that of quicksand--once an institution is entrapped, corrective efforts tend to add to the problem. For example, at the college level a vicious spiral may develop: decreasing

enrollments bring about reduced revenues; declining income results in cutbacks in programs, staff recruitment, and promotion; economy measures cause the institution to be less attractive to students, which in turn results in additional enrollment reductions.

From this pattern it might be concluded that if an institution is to remain healthy during the 1970's it is essential that effective methods of enrollment analysis and planning be developed--and that this be accomplished and operational before the institution becomes entrapped in the downward spiral.

II. A CASE STUDY IN DEMOGRAPHIC ANALYSIS AND PLANNING

The postsecondary enrollment dilemma and a positive approach to dealing with the problem can be illustrated by examining the situation in suburban Kansas City (Johnson County, Kansas) where school districts are making a concerted effort to generate realistic enrollment forecasts and to formulate practical long-range planning strategies. The enrollment trends, the causes and the associated problems and solutions for Johnson County are different only in degree from those of any other part of the country. Therefore, the approach described herein and the results reported may be useful to other districts which also need plans of action--not reaction--in order to remain, or to become, dynamic, thriving institutions at a time when many are struggling.

During the last decade educational enrollments in Johnson County, like the rest of the country, rose at an unprecedented rate. However, in the 1960's a downward trend began to develop and is currently intensifying. Birth rates were known to be the single most important contributing factor to this decline. Analysis indicates that the genesis of the problem was not low fertility rates in the 50's and 60's, but rather the low birth rates which followed the depression and preceeded World War II. This phenomenon, which is not atypical to the rest of the United States, Canada and Europe, is illustrated in Figure 1

below. Although the schematic oversimplifies the complex dynamics of demography, the following conclusions apply to many high schools and colleges throughout the country:

- The current decline in number of high school and college age students will intensify and reach a peak in the late 70's.
- The coming of this decline was predictable in the late 30's—some 40 years in advance.

Approach: Analytical Modeling

Johnson County Community College (JCCC) initiated the development of a long-range population planning capability in 1971. The project was jointly funded by the college and the Shawnee Mission K-12 public school district. The first step was the creation of a comprehensive data base for Johnson County. It included such information as births and deaths, population distribution by age and sex, census tract boundaries, housing units, land use zoning and school attendance rates. The accomplishment of this task provided a number of insights into the population profile and past trends. As important as these data were, however, they did not by themselves provide an integrated picture of the interrelationships that exist between and among pertinent

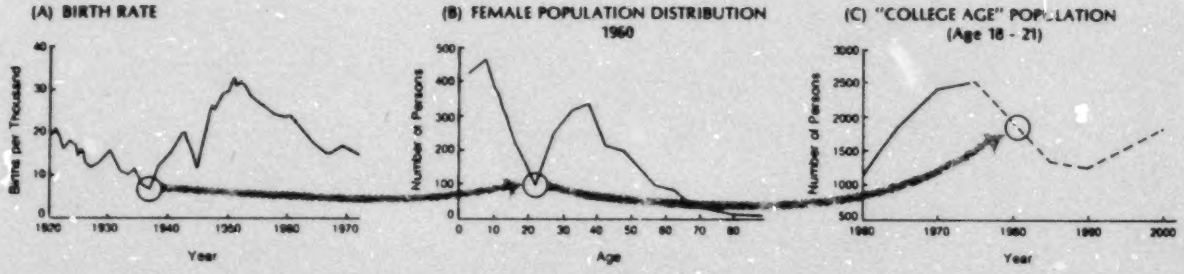


Figure 1. Population trends of a developed subdivision of Johnson County Kansas are representative of much of the nation. (A) Birth rates reached an all-time low during the late 30's; (B) this resulted in a relatively low number of women of child bearing age in 1960; (C) which, in turn, is projected to cause the current downturn in high school and college age population to be at its height during the late 70's.

planning variables, nor did they provide a means of assessing alternatives. To counter this deficiency, the college's research and planning staff developed a computerized planning model designed to translate the demographic data into a form amenable to decision-making.

Output from the demographic model consists of population projections of males and females by age group for each neighborhood or location in the county. Depending on directions specified by the user, a 30-year forecast for one geographic area within the county yields up to 6,000 individual projections. A county forecast for all areas may have as many as 200,000 individual projections.

One of the model's most useful features is its ability to provide for parametric analysis. Many parameters pertinent to enrollment analysis and planning are affected by socioeconomic and political conditions that cannot be known in advance. For example, fertility rate is a function of such factors as international relations, the state of the economy, abortion legislation and worldwide food shortages. Because the user can only speculate as to future fertility rates and because of the importance of this factor in demographic planning, the model makes it possible to analyze a range or series of possible values of this variable. Likewise there is provision for the systematic analysis of the population impact of all other significant parameters. Parametric studies of this type are sometimes referred to as "what if" analyses.

Simulation Process

In making a series of projections, the model begins by analytically profiling the county's quarter million residents. Each person's demographic characteristics--sex, age, place of residence--are recorded in the computer memory. The predicted lives of these persons are then simulated for the coming year. In each neighborhood and age group, some will die. Deaths are analytically simulated using actuarial rates as the basis for forecasting. Others are allowed

to give birth, taking into account prevalent fertility rates as they apply to the age of each of the potential mothers of Johnson County. Other residents will move out of the county and some will move to other neighborhoods within the county. Some will move into Johnson County neighborhoods from outside the county. These actions are simulated by employing current trends in migration propensity factors for each age group and neighborhood. Some areas within the county are allowed to grow to accommodate utilization of undeveloped land. Tracts which are zoned for park or industrial development are not allowed to accept in-migration.

The model uses an enhancement of cohort analysis. A "cohort" is defined as a group of people with a common characteristic. For example, women born in 1950 who live in Johnson County would be a cohort. People can leave this group only through death or through out-migration from Johnson County. For those born outside Johnson County in 1950, they can enter this group only through in-migration to Johnson County. This particular cohort of people can be traced throughout their entire life if death rates and net migration by age and sex are known.

The analytical simulation process is fundamentally straight forward. Referring to Figure 2, next page, assume that 375 women, 24 years of age in 1974 currently live in a specified neighborhood in Johnson County. Also assume that it is known that the following events will take place in the coming year; 15 will move out of the neighborhood, 32 will move in, 1 will die, and 38 will give birth. By simple arithmetic it can be predicted that one year later, the number of women, now 25 years of age, living in the neighborhood will be 391. The portion of the analysis that is not straight forward--the method of estimating the number of births, deaths and migrations--is accomplished through the generation and use of empirically derived mathematical algorithms.

The simulation process is continued one year at a time by allowing each age group to become one year older. The procedure is continued for as many years as the user specifies.

"What If" Variables

To enable the user to conduct parametric analysis, four major input variables are incorporated into the model: death rate, birth rate, net migration and land use.

Death Rates. Deaths are projected separately for males and females as a function of the number of people for each age. The user has two "what if" options. Using the term "nominal" rates to refer to the death rate equations of the model, the percentage of the "nominal" rate to be used in generating the forecast is specified for the beginning year and ending year of the projections. While the initial year multiplier would be 100 percent if the death equations are current, the initial year multiplier permits the use of an equation developed within the last few years. The ending year multiplier allows the user to examine the effect of declining or increasing death rates on the growth of the population.

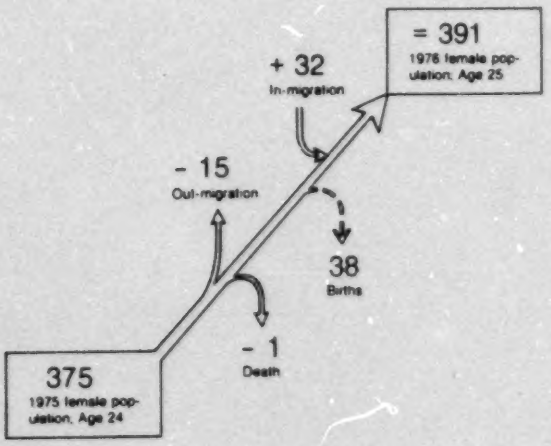


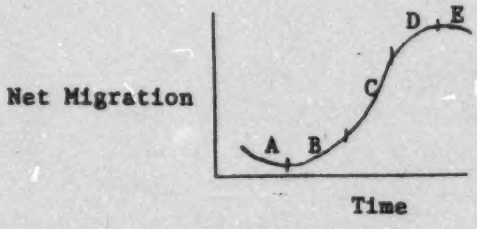
Figure 2. Sample calculation. Birth, death and migration data are obtained via empirically derived algorithms.

Birth Rates. Three options are available for projecting birth rates. The option typically used is the equation which projects the number of births using the fertility rates by age of women. The second option uses the crude birth rate. The third option uses a polynomial equation which can be modified by the user to provide various forecast relationships.

In addition to selecting one of the three birth rate equations, the user has two additional "what if" options. Using the term "nominal" rate to refer to the birth rates of the selected equation, the percentage of the "nominal" rate to be used in generating the forecast is specified for the beginning year and ending year of the projections. While the initial year multiplier would be 100 percent if the selected equation is current, the initial year multiplier is a means to use an equation developed within the last few years. The ending year multiplier permits the user to examine the effect of a declining or increasing birth rate upon the population projections. The multipliers for the intervening years are found by fitting a straight line through the two points determined by the initial year and multiplier together with the ending year and multiplier.

Net Migration. The user specifies the county net migration (number of people moving into minus number of people moving out) for the beginning year and ending year. Given the "what if" county net migration for a specified year, each geographic subdivision of the county (referred to as a subarea) receives a percentage of the county yearly net migration. Subareas are assigned a negative percentage if the net migrations can be expected to be negative. Total subarea net migration is assigned by age and sex. This age and sex distribution is accomplished by combining subareas of Johnson County into five groups on the basis of net migration for the preceding ten years. With net migration plotted as a function of time these five groups represent five stages in the development of land and are designated "A" through "E" on the typical

population S-curve below.



These five stages may be described as follows:

A. Agricultural - undeveloped in terms of potential population capacity. During this stage there is a net out-migration typical of most rural areas today.

B. Initial transition from rural to urban/suburban accompanied by a shift from out-migration to in-migration.

C. Maximum rate of development. During this stage in-migration greatly exceeds out-migration.

D. Reduced rate of growth. As the area approaches maturity the rate of net in-migration begins to decline.

E. Maximum development. The available land is essentially fully utilized. During this stage, there is a gradual shift from in-migration to out-migration.

Land Use. Land use designations are based on zoning. Categories employed by the model are special, agriculture, industrial, commercial, low density residential, medium density residential, high density residential and unzoned. The zoning is specified for the beginning year. The user has the option of incorporating anticipated zoning changes into the model. This information can be obtained from city or county zoning boards, regional councils, title insurance companies, etc. Each subarea is divided into smaller subunits. Depending upon the population density of the subarea, these smaller subunits are sections (approximately one square mile), ¼ sections (approximately ¼ square mile) or ¼¼ sections (approximately 1/16 square mile). For each of these subunits, zoning is specified. The portion of the subunit which lies within the boundaries of the subarea is

recorded as a fraction. The capacity (maximum number of people who could ultimately reside in the subunit on the basis of current zoning) is calculated and incorporated into the population projections.

Modified Cohort Analysis

The model has two major differences from the typical cohort survival model. Cohort analysis for a population living within a fixed geographic area does not directly consider the population affects of zoning or non-constant rates of migration. For example, suppose a subarea of the county has a 1970 population of 20,000. The typical cohort analysis might project a population of 30,000 in 1980 and a population of 60,000 in 2000. However, if the land is primarily zoned for low density residential, a maximum realistic population might be 50,000. The basic problem is related to estimates of net migration. To alleviate the problem and prevent a geographic area from exceeding its capacity, a two-point logistic curve (see Section III) is used to develop subarea net migration percentages for the ending year. The logistic curve is also used to distribute the population of each subarea to smaller geographic units such as sections.

In summary, the model can be described in general terms as being a modified cohort survival model. Regression analysis is used to develop equations for birth rates and death rates. The two-point logistic curve is used to develop net migration subarea percentages and for distributing the population of each subarea to smaller units such as sections.

Output

A computer printout mock-up is shown in Appendix A. For each identified subarea, the user specifies the year increment for printout. For example, data can be printed for each year, every three years, every five years, or in general every n years where n is an integer. If the starting year is 1973 and n is four, printouts will be for 1977, 1981, etc. The user also specifies the

age increment for printout. If the selected increment is five, the printout will give the forecast for ages 0-4, 5-9, 10-14, etc.

For each year of printout, the population is presented using the following age code. When the printout is by five-year age groups, the "age 5" denotes children less than five years old, while "age 10" denotes children five through nine, and "15" denotes children age ten through fourteen. When the printout is by one-year ages, the "age 1" denotes births, while "age 2" denotes children age one, "age 3" denotes children age two, etc. The printed ages are, therefore, used as upper limits to the actual age interval. Corresponding migration and death output are applicable to the preceding year.

In addition, for each year of printout, population may be optionally presented by ¼¼ section (or other subunit) for the following age groups:

- Under five
- Five through eight
- Nine through eleven
- Twelve through fourteen
- Fifteen through seventeen
- Eighteen and nineteen
- Twenty through twenty-four
- Twenty-five through forty
- Forty-one through fifty-nine
- Sixty and over

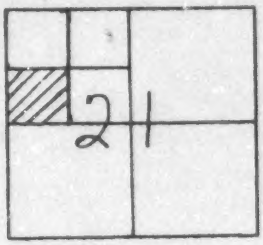
The population for these ¼¼ sections (sixteen ¼¼ sections = one section) should be considered in aggregation with surrounding ¼¼ sections. This aggregation is especially important in areas which currently are sparsely populated or where zoning changes seem likely. These ¼¼ section populations should be used individually only as guides showing future population trends.

The approximate 1960 and 1973 populations are shown in the mock-up printout on page 12 for each ¼¼ section along with their capacities. The relationships of the 1973 population to the capacity is presented as a decimal "FC" (1973 population/capacity). Projections by ¼¼ section continue to present the relationships of projected population to capacity as a decimal under the heading "Pop./Cap."

The ¼¼ sections are coded as follows:

- The first and second digits refer to the township
- The third and fourth digits refer to the range
- The fifth and sixth digits refer to the section
- The seventh digit is the ¼ section (beginning in the upper right hand quadrant with the number 1 and moving counter clockwise to the upper left hand quadrant denoted 2, the lower left quadrant denoted 3 and lower right quadrant denoted 4)
- The eighth digit is the ¼¼ section (designated by the same procedure used for ¼ sections).

For example, the digits 12252123 designate township 12, range 25, section 21, upper left hand quadrant, lower left ¼¼ section. This ¼¼ section is shaded in the sample section 21 below.



The mock-up sample on the following page has a 0 as the eighth digit since the level of detail is only ¼ section. The program as applied to Johnson County analyzed three major sets of assumptions. They are summarized below:

- Low Growth Assumption. For net migration of 1200 per year, the printout is by five-year age increments for males and females for 1978, 1983, 1988, 1993 and 1998. The fertility rates by age of women were permitted to decline so that by the year 2001 they will be 80 percent of the 1973 fertility rates.
- Nominal Growth Assumption. For net migration of 3000 per year, the printout is by one-year age increments for males and females for 1974-1984. Fertility rates by age of women were assumed to continue at the current rates.
- High Growth Assumption. For net migration of 5000 per year, the printout is by five-year age increments for males and females for 1978, 1984, 1988, 1993 and 1998. Fertility rates were assumed to gradually increase to the 1970 fertility rates.

SAMPLE PRINTOUT INCORPORATING SUB-UNIT PROJECTIONS

CAPACITY/AREA = 0. 256. 256. 0. 0. 1200. 2400. 4000.

SUB AREA = TEST STARTING YR. = 1973. TOT. POP. = 5391. DFC/DT = -.005000 FC = 0.820500

TCUR = 1973. TBASE = 1960. SUB A = -0.19610739E 01 SUB B = 0.33948969E-01

BIRTH RATE WHAT IF? (FRAC1,FRAC2) = 0.70 0.70 BIRTH RATE TYPE USED = 1 DEATH RATE WHAT IF? (FR1,FR2) = 1.00 1.00

ID	POP60	POP73	CAP.	FC	DFC/DT	PF		A	B
12251530	7.0	5.0	5.0	0.9999990	0.0000000	0.00060	-0.14243654E	02	0.33948969E-01
12251630	602.0	740.0	1176.0	0.6343538	0.0094192	1.16980	-0.99227750E	00	0.33948969E-01
12251640	983.5	975.0	1223.0	0.7971941	0.0000000	0.00000	-0.18101845E	01	0.33948969E-01
12252110	2058.0	1937.0	1992.0	0.9723896	0.0000000	0.00000	-0.58240337E	01	0.17403603E 00
12252120	2058.0	1728.0	2352.0	0.7346939	0.0000000	0.00000	-0.14599056E	01	0.33948969E-01

YEAR = 1978 SUB AREA = TEST

AGE	MALE POP.	MALE MIG.	MALE DEA.	FEM. POP.	FEM. MIG.	FEM. DEA.
5	205.1	0.9	0.9	194.8	0.5	0.6
10	243.6	-4.3	0.1	245.1	-5.2	0.1
15	251.2	-9.7	0.1	211.0	-8.6	0.1
20	250.6	-9.2	0.1	246.4	-5.9	0.1
25	238.0	6.1	0.1	227.5	9.1	0.1
30	243.1	9.4	0.2	257.2	6.5	0.1
35	276.2	0.3	0.3	300.0	-2.9	0.2
40	190.7	-7.6	0.3	166.1	-7.6	0.2
45	106.3	-2.4	0.3	116.5	-2.1	0.2
50	101.8	-1.7	0.5	124.1	-1.9	0.3
55	119.2	-2.7	0.9	138.9	-1.2	0.6
60	112.5	-0.7	1.4	148.3	-0.1	1.1
65	106.1	-0.8	2.1	137.6	-1.0	1.7
70	85.7	-0.7	2.7	103.9	-0.6	2.0
75	64.2	-0.2	3.0	63.7	0.0	2.1
80	27.5	0.0	2.0	42.7	-0.2	2.3
85	17.1	0.5	1.8	23.7	0.7	1.9
90	7.7	0.0	1.0	10.8	-0.5	1.3
95	1.3	0.0	0.2	5.4	0.0	0.9
100	0.4	0.0	0.1	2.5	0.0	0.4
105	0.0	0.0	0.0	0.0	0.0	0.0
110	0.0	0.0	0.0	0.0	0.0	0.0
0	2657.3	-22.8	18.1	2766.1	-20.8	16.3

T R S1/4	POP.	POP./CAP.	CAPACITY	1	5	9	12	15	18	20	25	41	60
12251530	5.21	1.04120	5.00	0.38	0.28	0.37	0.3	0.3	0.2	0.4	1.42	0.89	0.67
12251630	737.51	0.62714	1176.00	54.38	39.87	51.72	37.6	39.5	27.6	62.9	202.01	126.70	95.23
12251640	986.08	0.80626	1223.04	72.71	53.31	69.16	50.2	52.8	36.9	84.0	270.57	169.10	127.33
12252110	1962.31	0.98509	1992.00	144.69	106.09	137.62	99.9	105.1	73.4	167.2	536.96	337.97	253.38
12252120	1732.35	0.73654	2352.00	127.74	93.66	121.49	88.2	92.8	64.8	147.7	473.27	298.58	224.11
0	5423.46	0.80371	6748.04	5208.84									

Hypothetical Illustration

Assume a fictitious county called Pirate County had a 1970 population of 300,000. The age and sex distribution is known for 1970. Equations have been developed for estimating births, deaths and net migration. The number of two year old males in 1971 is obtained by adding the number of one year old males in 1971 to the estimated net number of one year old males who moved into the county during 1971 and subtracting the estimated number of one year old male deaths for 1971. The procedure is similar for all ages and each year. The sample printouts in Appendix A illustrate the impact of some alternative user assumptions for population growth in Pirate County.

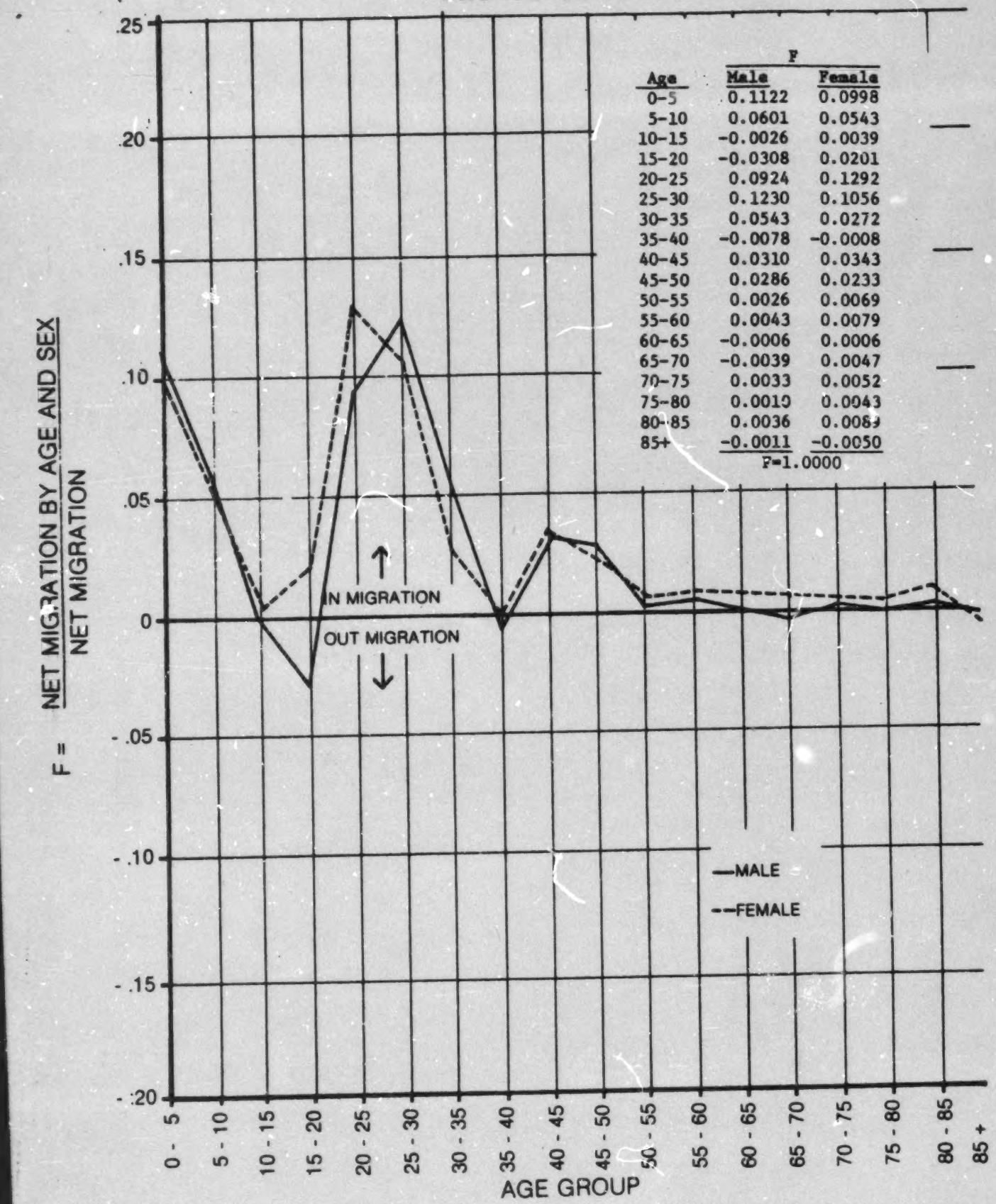
Illustration of Results

The county can be characterized as a growing community with a positive net migration and with the number of births approximately three times the number of deaths. The table below summarizes the population projections for the year 1985 and the year 2000 under each of eight user alternative assumptions. The graph on the following page characterizes net migration by age group and sex. The printouts in Appendix A present the sample assumptions together with the corresponding projections for 1985 and 2000.

PIRATE COUNTY POPULATION PROJECTIONS
1970 Population was 300,000

User Alternative Assumptions	1985 Population	2000 Population
Zero Net Migration	363,616	414,249
Low Birth Rate and Net Migration	376,851	433,345
Low Net Migration	381,678	454,718
Low Birth Rate	412,561	511,139
Nominal	417,800	535,655
High Birth Rate	423,040	560,399
High Net Migration	453,924	616,594
High Birth Rate and Net Migration	459,576	644,495

PIRATE COUNTY



Age	F Male	F Female
0-5	0.1122	0.0998
5-10	0.0601	0.0543
10-15	-0.0026	0.0039
15-20	-0.0308	0.0201
20-25	0.0924	0.1292
25-30	0.1230	0.1056
30-35	0.0543	0.0272
35-40	-0.0078	-0.0008
40-45	0.0310	0.0343
45-50	0.0286	0.0233
50-55	0.0026	0.0069
55-60	0.0043	0.0079
60-65	-0.0006	0.0006
65-70	-0.0039	0.0047
70-75	0.0033	0.0052
75-80	0.0019	0.0043
80-85	0.0036	0.0089
85+	-0.0011	-0.0050
	F=1.0000	

III. MISCELLANEOUS THEORY

TWO-POINT LINEAR REGRESSION

$$Y = bX + c$$

Given two points (x_1,y_1) and (x_2,y_2) the equation is

$$Y = \frac{y_2-y_1}{x_2-x_1} (X - x_1) + y_1$$

Example

Using 1960 and 1970 Pirate County population

Y = Projected population for year X

y_2 = 1970 population

y_1 = 1960 population

$$Y = \frac{300,000-180,000}{1970-1960} (X - 1960) + 180,000$$

$$Y = 12,000 (X - 1960) + 180,000$$

If X = 1980, Y = 420,000

X = 2000, Y = 660,000

MULTIPLE-POINT LINEAR REGRESSION

$$Y = bX + a$$

where Y is criterion

X is predictor variable

Given n pairs of observations for X and Y,

$$b = \frac{n \sum_{i=1}^{n} X Y - \sum_{i=1}^{n} X \sum_{i=1}^{n} Y}{n \sum_{i=1}^{n} X^2 - \left(\sum_{i=1}^{n} X\right)^2}, \quad a = \bar{Y} - b\bar{X} = \frac{\sum_{i=1}^{n} Y}{n} - b\left(\frac{\sum_{i=1}^{n} X}{n}\right)$$

Example

Using the population data for Pirate County, 1920-1970, the criterion Y is population while the predictor variable X is year,

$n = 6$, $\sum_{i=1}^{6} X = 1920+1930+1940+1950+1960+1970=11670$

$$\sum_{i=1}^{6} Y = 18000 + 30000 + 39000 + 75000 + 180000 + 300000 = 642000$$

$$\sum_{i=1}^{6} X^2 = (1920)^2+(1930)^2+(1940)^2+(1950)^2+(1960)^2+(1970)^2=22,699,900$$

$$\sum_{i=1}^{6} X Y = (1920)(18,000)+(1930)(30,000)+(1940)(39,000)+(1950)(75,000) + (1960)(180,000)+(1970)(300,000)=1,258,170,000$$

$$b = \frac{6(1,258,170,000)-(11670)(642,000)}{6(22,699,900)-(11670)^2} = \frac{5688}{1.05} = 5417.14$$

$$a = \frac{642,000}{6} - 5417.14\left(\frac{11670}{6}\right) = -10,429,337.30$$

$$Y = 5417.14(X) - 10,429,337.30$$

For X = 1980, Y = 296,600 which is less than actual 1970 population.

TWO-POINT LOGISTIC CURVE

S - shaped

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

Where P is population

t_1 is starting year

θ is number of years past starting year

K is holding capacity

Given K and P_{t_1} for year t_1

P_{t_2} for year t_2, where $t_2 > t_1$

$$a = \ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right) \qquad b = \left[\frac{\ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a}{t_2-t_1}\right]$$

Example

$K = 1{,}500{,}000$

$t_1 = 1960$, $P_{1960} = 180{,}000$

$t_2 = 1970$, $P_{1970} = P_{1960+10} = 300{,}000$

$$a = \ln\left(\frac{1{,}500{,}000-180{,}000}{180{,}000}\right) = \ln\left(\frac{1{,}320{,}000}{180{,}000}\right) = \ln\left(\frac{132}{18}\right)$$

$$= \ln\left(\frac{22}{3}\right) = \ln(22) - \ln(3) = 3.09104 - 1.09861 = 1.99243$$

$$b = \frac{\ln\left(\frac{1{,}500{,}000-300{,}000}{300{,}000}\right) - 1.99243}{1970-1960} = \frac{\ln\left(\frac{1{,}200{,}000}{300{,}000}\right) - 1.99243}{10}$$

$$= \frac{\ln(4) - 1.99243}{10} = \frac{1.38629-1.99243}{10} = \frac{-.60614}{10} = -.060614$$

So $$P_{1960+\theta} = \frac{1{,}500{,}000}{1+e^{1.99243-.0606140\theta}}$$

Proof that a and b are obtained as specified on preceeding page for the logistic curve.

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

At time t_1, $\theta = 0$, $P_{t_1} = \frac{K}{1+e^{a}}$ so $P_{t_1}(1+e^{a}) = K$

$$P_{t_1} + P_{t_1}(e^{a}) = K$$

$$P_{t_1} e^{a} = K - P_{t_1}$$

$$e^{a} = \frac{K-P_{t_1}}{P_{t_1}}$$

$$a = \ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right)$$

At time t_2, $\theta = t_2-t_1$ where $t_2>t_1$

$$P_{t_2} = \frac{K}{1+e^{a+b(t_2-t_1)}}$$

$$P_{t_2}\left(1+e^{a+b(t_2-t_1)}\right) = K$$

$$P_{t_2} + P_{t_2} e^{a+b(t_2-t_1)} = K$$

$$P_{t_2} e^{a+b(t_2-t_1)} = K-P_{t_2}$$

$$e^{a+b(t_2-t_1)} = \frac{K-P_{t_2}}{P_{t_2}}$$

$$a+b(t_2-t_1) = \ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right)$$

So $b(t_2-t_1) = \ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a$

And $$b = \frac{\ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right) - a}{t_2-t_1}$$

23

MULTIPLE-POINT LOGISTIC CURVE

$$P_{t_1+\theta} = \frac{K}{1+e^{a+b\theta}}$$

Given year t_1, capacity K and n observations, a and b are found so that the "best fit" is found. The procedure utilizes a transformation as follows:

$$P_{t_1+\theta}(1+e^{a+b\theta}) = K$$

$$P_{t_1+\theta} + P_{t_1+\theta}e^{a+b\theta} = K$$

$$P_{t_1+\theta}e^{a+b\theta} = K - P_{t_1+\theta}$$

$$e^{a+b\theta} = (K - P_{t_1+\theta})\frac{1}{P_{t_1+\theta}}$$

$$a+b\theta = \ln\left(\frac{K-P_{t_1+\theta}}{P_{t_1+\theta}}\right)$$

Let $Z = \ln\left(\frac{K-P_{t_1+\theta}}{P_{t_1+\theta}}\right)$

Then $Z = a + b\theta$

Next find the best linear fit for $Z = a + b\theta$, where $\theta = t_2 - t_1$

Example

Use the 1950, 1960 and 1970 Pirate County population data* together with a capacity of 1,500,000 people and a base year t_1=1960

For 1950, $\theta = -10$, $Z = \ln\left(\frac{1,500,000-75,000}{75,000}\right) = 2.94444$

1960, $\theta = 0$, $Z = \ln\left(\frac{1,500,000-180,000}{180,000}\right) = 1.99243$

1970, $\theta = 10$, $Z = \ln\left(\frac{1,500,000-300,000}{300,000}\right) = 1.38629$

* $P_{1950} = 75,000$

$P_{1960} = 180,000$

$P_{1970} = 300,000$

$$\sum_{i=1}^{3} \theta = -10 + 0 + 10 = 0$$

$$\sum_{i=1}^{3} Z = 2.94444 + 1.99243 + 1.38629 = 6.32316$$

$$\sum_{i=1}^{3} \theta^2 = 100 + 0 + 100 = 200$$

$$\sum_{i=1}^{3} \theta Z = (-10)(2.94444) + 0(1.99243) + 10(1.38629) = -15.5815$$

$$b = \frac{3(-15.5815) - (0)(6.32316)}{3\ (200) - (0)^2} = -.07791$$

$$a = \frac{6.32316}{3} - (-.07791)\left(\frac{0}{3}\right) = 2.10772$$

So $Z = 2.10772 - .077910$

Or $P_{t_1+\theta} = \frac{K}{1+e^{2.10772-.077910}}$ for $K = 1,500,000$

For 1980, $\theta = 20$

$$P_{1980} = \frac{1,500,000}{1+e^{2.10772-1.55820}} = \frac{1,500,000}{1+e^{.54952}}$$

$$= \frac{1,500,000}{1+1.7324} = \frac{1,500,000}{2.7324} = 548,968$$

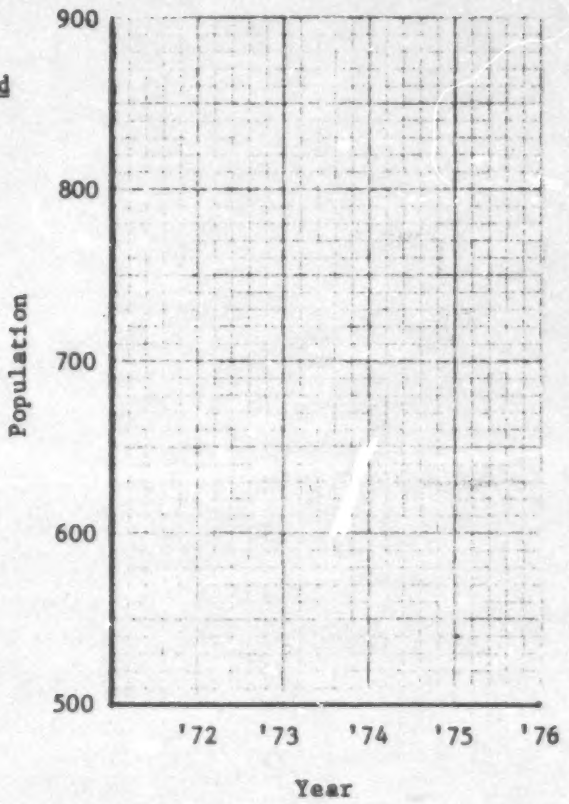
IV. PRACTICE PROBLEMS

PRACTICE PROBLEM: REGRESSION AND COHORT SURVIVAL METHODOLOGY

Problem: Estimate the 1976 population by age group using regression and cohort survival methods of analysis. The population data base for the years 1972-75 is given in the table below:

Age	Actual '72	Actual '73	Actual '74	Actual '75	Regress. '76	Cohort '76
6	690	660	600	560		
7	720	718	693	636		
8	747	734	740	721		
9	760	755	749	762		
10	703	775	778	779		
11	656	724	806	817		
12	560	663	738	830		

Regression Method



Sample Cohort Survival Projection #1

$a = 6$, $a+1 = 7$

$$\frac{P(a+1,\text{'73})}{P(a,\text{'72})} = \frac{P(\quad,\text{'73})}{P(\quad,\text{'72})} = \frac{(\quad)}{(\quad)} = (\quad) \ ①$$

$$\frac{P(a+1,\text{'74})}{P(a,\text{'73})} = \frac{P(\quad,\text{'74})}{P(\quad,\text{'73})} = \frac{(\quad)}{(\quad)} = (\quad) \ ②$$

$$\frac{P(a+1,\text{'75})}{P(a,\text{'74})} = \frac{P(\quad,\text{'75})}{P(\quad,\text{'74})} = \frac{(\quad)}{(\quad)} = (\quad) \ ③$$

$$\frac{① + ② + ③}{3} = \frac{(\quad) + (\quad) + (\quad)}{3} = \frac{(\quad)}{3} = (\quad) \ ④$$

So $P(\quad,\text{'76}) = ④ \times P(\quad,\text{'75}) = (\quad) \times (\quad) =$ ☐

Sample Cohort Survival Projection #2

$a = 6$, $a+1 = 7$

$$\frac{P(a+1,\text{'73})}{P(a,\text{'72})} = \frac{P(\quad,\text{'73})}{P(\quad,\text{'72})} = \frac{(\quad)}{(\quad)} = (\quad) \ ①$$

$$\frac{P(a+1,\text{'74})}{P(a,\text{'73})} = \frac{P(\quad,\text{'74})}{P(\quad,\text{'73})} = \frac{(\quad)}{(\quad)} = (\quad) \ ②$$

$$\frac{P(a+1,\text{'75})}{P(a,\text{'74})} = \frac{P(\quad,\text{'75})}{P(\quad,\text{'74})} = \frac{(\quad)}{(\quad)} = (\quad) \ ③$$

$$\frac{① + ② + ③}{3} = \frac{(\quad) + (\quad) + (\quad)}{3} = \frac{(\quad)}{3} = (\quad) \ ④$$

So $P(\quad,\text{'76}) = ④ \times P(\quad,\text{'75}) = (\quad) \times (\quad) =$ ☐

PRACTICE PROBLEM: TWO-POINT LOGISTIC METHOD

Problem: Forecast the population living within the boundaries of a urban-suburban school district for various land use assumptions using the two-point logistic method of analysis for the years 1980, 1990, 2000:

P_{1960} = 180,000 P_{1970} = 300,000 Area = 400 sq. mi.

Land Use Assumption:

Zoning	Sq.Miles	Ultimate Density Pop/Sq.Mile	Holding Capacity
R_1: Low Density Residential		4,795	_____
R_2: Medium Density Residential		12,787	_____
R_3: High Density Residential		25,574	_____
Agricultural		1,066	_____
Parks, Commercial, Industrial		0	_____
		K = Total Holding Capacity =	

$$P_{1960+\theta} = \frac{K}{1+e^{\left\{\ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right)+\left[\frac{\ln\left(\frac{K-P_{t_2}}{P_{t_2}}\right)-\ln\left(\frac{K-P_{t_1}}{P_{t_1}}\right)}{t_2-t_1}\right]\right\}\theta}}$$

Where θ = 1980 - 1960 = 20

P_{t1} = 180,000

P_{t2} = 300,000

t_2-t_1 = 1970-1960 = 10

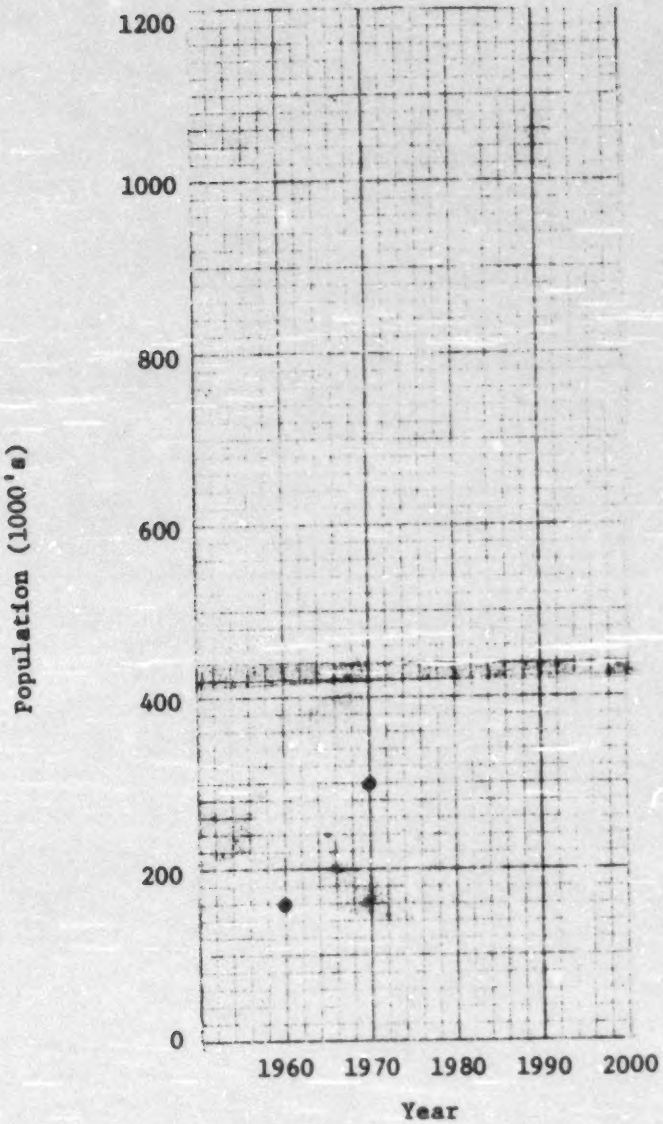
K = _____

Solving the above equation yields the following results:

Land Use K	P_{1980}
350,000	340,000
600,000	420,000
4,100,000	490,000

So $P_{1980} \approx$ [ ]

Two-Point Logistic Method



APPENDIX A

SAMPLE PRINTOUTS

ASSUMPTION:
ZERO NET MIGRATION

```
 10/22/74. 10.18.37.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHRPIR,1970,.001,2000,.001
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE  M POP  M MIG  M DEA  F POP  F MIG  F DEA
 10  33578      0     85  32032      0     59
 20  27381     -0     14  26076      0     10
 30  34348      0     23  33272      0     19
 40  21604      0     27  24103      0     25
 50  20232      0     67  22214      0     55
 60  18330      0    173  19835      0    137
 70  15256     -0    351  16197      0    287
 80   6528      0    358   7790      0    368
 90   1557      0    170   2504      0    277
100    214      0     36    489      0     93
110     22      0      4     66      0     12
  0 179049      0   1307 184567      0   1342
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE  M POP  M MIG  M DEA  F POP  F MIG  F DEA
 10  33624      0     78  32058      0     60
 20  36020     -0     18  34389      0     18
 30  29274      0     19  27978      0     22
 40  30949      0     42  29775      0     49
 50  30174      0     87  29660      0    102
 60  17178      0    157  21103      0    211
 70  15740     -0    363  16735      0    465
 80  11439      0    628  11841      0    865
 90   3699      0    370   3385      0    529
100    514      0     76    348      0     95
110     42      0      6     24      0      7
  0 207953      0   1844 206296      0   2424
 STOP.
```

ASSUMPTION:
NET MIGRATION DECREASES 67 PERCENT
BIRTH RATES DECREASE 20 PERCENT BY 2000

```
 10/22/74. 09.54.01.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,.8,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHRPIR,1970,1000,2000,1000
YEAR =  1985
      1985   1984   1984   1985   1984   1984
AGE   M POP  M MIG  M DEA  F POP  F MIG  F DEA
 10   33715    172     82  32108    154     57
 20   28802    -33     15  27490     24     10
 30   35091    215     24  34779    235     20
 40   23829     47     30  26429     26     28
 50   21215     60     70  23019     58     56
 60   18843      7    178  20387     15    140
 70   15376     -5    353  16397      5    290
 80    6526      4    358   7879     10    373
 90    1583      3    173   2592      4    286
100     217      0     36    490      0     93
110      21      0      4     64      0     12
  0  185219    470   1321 191632    530   1366
YEAR =  2000
      2000   1999   1999   2000   1999   1999
AGE   M POP  M MIG  M DEA  F POP  F MIG  F DEA
 10   31627    172     71  30098    154     54
 20   35962    -33     18  34375     24     19
 30   31403    215     20  30729    235     25
 40   33223     47     46  32249     26     54
 50   32747     60     96  32650     58    113
 60   19378      7    173  23114     15    227
 70   16367     -5    376  17418      5    483
 80   11666      4    637  12183     10    886
 90    3732      3    374   3488      4    545
100     518      0     76    352      0     97
110      42      0      6     24      0      7
  0  216666    470   1893 216679    530   2511
STOP.
```

ASSUMPTION:
NET MIGRATION DECREASES 67 PERCENT

```
 10/22/74. 09.49.23.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FI.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHPRIP,1970,1000,2000,1000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE  M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10  35994     172      88   34280     154      62
 20  28994     -33      15   27673      24      10
 30  35091     215      24   34779     235      20
 40  23829      47      37   24429      24      28
 50  21215      60      70   23018      55      56
 60  18843       7     178   20057      15     140
 70  15376      -5     353   16397       5     290
 80   4526       4     358    7579      10     373
 90   1583       3     173    2592       4     286
100    217       0      36     490       0      93
110     21       0       4      64       0      12
  0 187690     470    1328 193988     530    1372
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE  M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10  37920     172      88   36097     154      67
 20  39457     -33      19   37902      24      20
 30  33354     215      21   31437     235      25
 40  33223      47      46   32249      26      54
 50  32747      60      96   32650      53     113
 60  19378       7     173   23114      15     227
 70  16367      -5     376   17418       5     483
 80  11666       4     637   12183      10     886
 90   3732       3     374    3488       4     545
100    518       0      76     352       0      97
110     42       0       4      24       0       7
  0 227604     470    1919 227114     530    2524
STOP.
```

ASSUMPTION:
BIRTH RATES DECREASE 20 PERCENT BY 2000

```
 10/29/74. 09.35.23.
PROGRAM    COHORT


ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,.8,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHRPIR,1970,3000,2000,3000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   33344     517      91   36410     469      63
 20   32020    -100      16   30676      72      11
 30   36576     646      25   37794     704      21
 40   28280     140      35   31079      79      32
 50   23181     179      75   24624     173      59
 60   19969      21     186   21490      44     147
 70   15617     -14     357   16798      14     296
 80    6522      13     357    8077      29     383
 90    1637       8     180    2768      13     306
100     222       0      37     493       0      93
110      21       0       4      61       0      11
  0  202289    1405    1362  219272    1591    1424
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   39075     517      87   37086     469      66
 20   42838    -100      21   41021      72      22
 30   37496     646      24   37982     704      30
 40   39171     140      53   39196      79      44
 50   37893     179     113   38632     173     135
 60   23777      21     205   27134      44     259
 70   17620     -14     402   18733      14     520
 80   12120      13     657   12866      29     930
 90    3798       8     381    3695      13     579
100     527       0      79     360       0      99
110      44       0       7      25       0       9
  0  254359    1409    2026  256780    1591    2713
 STOP.
```

ASSUMPTION:
NOMINAL ASSUMPTIONS

```
 10/22/74. 09.27.47.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHPBIR,1970,3000,2000,3000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   40826     517      98   38777     462      69
 20   32219    -100      14   30867      72      11
 30   36576     646      25   37794     704      21
 40   28280     140      35   31079      79      32
 50   23181     179      75   24626     173      59
 60   19869      21     186   21490      44     147
 70   15617     -14     357   16798      16     296
 80    6522      13     357    8077      29     383
 90    1637       8     180    2768      12     306
100     222       0      37     493       0      93
110      21       0       4      61       0      11
  0 204971    1409    1370 212829    1591    1430
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   46511     517     107   44175     462      82
 20   46932    -100      23   44929      72      24
 30   38514     646      25   38954     704      31
 40   39171     140      53   39196      79      44
 50   37893     179     113   38632     173     135
 60   23777      21     205   27134      44     259
 70   17620     -14     402   18783      16     520
 80   12120      13     657   12866      29     930
 90    3798       8     381    3696      12     579
100     527       0      78     360       0      99
110      44       0       7      25       0       8
  0 266906    1409    2049 268749    1591    2730
STOP.
```

ASSUMPTION:
BIRTH RATES INCREASE 20 PERCENT BY 2000

```
 10/22/74. 09.58.20.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1.2,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHRPIR,1970,3000,2000,3000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   43308     517     106   41144     462      74
 20   32419    -100      16   31058      72      11
 30   36576     646      25   37794     704      21
 40   28280     140      35   31079      79      32
 50   23181     179      75   24626     173      59
 60   19869      21     186   21490      44     147
 70   15617     -14     357   16798      16     296
 80    6522      13     357    8077      29     383
 90    1637       8     180    2768      12     306
100     222       0      37     493       0      93
110      21       0       4      61       0      11
  0  207653    1409    1378  215387    1591    1435
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   54064     517     127   51375     462      97
 20   51025    -100      25   48837      72      26
 30   39531     646      25   39927     704      31
 40   39171     140      53   39196      79      64
 50   37893     179     113   38632     173     135
 60   23777      21     205   27134      44     259
 70   17620     -14     402   18783      16     520
 80   12120      13     657   12866      29     930
 90    3798       8     381    3696      12     579
100     527       0      78     360       0      99
110      44       0       7      25       0       8
  0  279570    1409    2072  280829    1591    2748
 STOP.
```

ASSUMPTION:
NET MIGRATION INCREASES 67 PERCENT

```
 10/22/74. 10.03.57.
PROGRAM   COHORT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTION) TO BE APPLIED TO BIRTH RATES & TYPE
 ? 2000,1,1
ENTER YR & %(FRACTION) TO BE APPLIED TO DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? COHRPIR,1970,5000,2000,5000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   45658     862     108   43274     771      75
 20   35445    -167      18   34060     120      12
 30   38062    1077      26   40810    1174      23
 40   32731     233      41   35729     132      37
 50   25148     298      80   26234     288      63
 60   20895      35     195   22593      74     154
 70   15858     -23     360   17198      27     302
 80    6518      22     357    8275      48     393
 90    1690      13     187    2944      20     325
100     227       0      37     495       0      92
110      20       0       3      59       0      11
  0  222253    2348    1412  231671    2652    1488
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE   M POP   M MIG   M DEA   F POP   F MIG   F DEA
 10   55103     862     126   52254     771      96
 20   54206    -167      26   51955     120      28
 30   44673    1077      28   46272    1174      37
 40   45119     233      60   46143     132      75
 50   43038     298     130   44613     288     158
 60   28176      35     237   31154      74     292
 70   18874     -23     428   20147      27     556
 80   12574      22     676   13550      48     973
 90    3864      13     388    3903      20     612
100     536       0      79     368       0     102
110      45       0       7      25       0       8
  0  306209    2348    2186  310385    2652    2935
 STOP.
```

ASSUMPTION:
BIRTH RATES INCREASE 20 PERCENT BY 2000
NET MIGRATION INCREASES 67 PERCENT

```
 10/22/74. 10.11.56.
PRØGRAM   CØHØRT

ENTER STARTING & ENDING YEAR, PRINT DELTA YRS,& PRT. AGE INTERVALS
 ? 1970,2000,15,10
ENTER YR, %(FRACTIØN) TØ BE APPLIED TØ BIRTH RATES & TYPE
 ? 2000,1.2,1
ENTER YR & %(FRACTIØN) TØ BE APPLIED TØ DEATH RATES
 ? 2000,1
ENTER MIG.& START P FL.NAME,YR1,NET MIG1,YR2,NET MIG2
 ? CØHRPIR,1970,5000,2000,5000
YEAR =  1985
      1985    1984    1984    1985    1984    1984
AGE  M PØP   M MIG   M DEA   F PØP   F MIG   F DEA
 10   48344     862     116   45835     771      81
 20   35653    -167      18   34259     120      12
 30   38062    1077      26   40810    1174      23
 40   32731     233      41   35729     132      37
 50   25148     298      80   26234     288      63
 60   20895      35     195   22593      74     154
 70   15858     -23     360   17198      27     302
 80    6518      22     357    8275      48     393
 90    1690      13     187    2944      20     325
100     227       0      37     495       0      92
110      20       0       3      59       0      11
  0  225146    2348    1421  234430    2652    1494
YEAR =  2000
      2000    1999    1999    2000    1999    1999
AGE  M PØP   M MIG   M DEA   F PØP   F MIG   F DEA
 10   63806     862     150   60550     771     114
 20   58697    -167      28   56243     120      30
 30   45758    1077      29   47309    1174      37
 40   45119     233      60   46143     132      75
 50   43038     298     130   44613     288     158
 60   28176      35     237   31154      74     292
 70   18874     -23     428   20147      27     556
 80   12574      22     676   13550      48     973
 90    3864      13     388    3903      20     612
100     536       0      79     368       0     102
110      45       0       7      25       0       8
  0  320489    2348    2212  324006    2652    2956
STØP.
```

UNIVERSITY OF CALIF.
LOS ANGELES

JAN 30 1976

CLEARINGHOUSE FOR
JUNIOR COLLEGES